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MODIFIED STRENGTH OF MATERIALS AND ENERGY APPROACH TO DETERMINE EFFECTIVE PROPERTIES OF PIEZOELECTRIC FIBER REINFORCED COMPOSITE

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ABSTRACT

In this paper, modified strength of materials (MSM) and strain energy-based approach have been used to determine effective electro-mechanical properties of piezoelectric fiber reinforced composites (PFRC). Based upon these methods, micromechanics model has been formulated using a rectangular representative volume element (RVE) which contains both fiber as well as matrix phases. Result obtained has been compared with strength of materials (SM) approach to the similar problems available in literatures. Results suggest that there is wide variation in estimation of properties obtained through strength of materials method to the methods adopted in this work. Strength of materials (SM) method in most of cases overestimates or underestimates the actual result. Though each of such approaches has its own limitations, the adopted method in this paper refines result obtained through strength of materials (SM), as it takes into account of the several conditions, which has not been dealt while analysing with strength of materials method. Both modified strength of materials and energy approach reduces aberration generated in result, due to generation of unequal strain in matrix and fiber phases as well as Poisson's ratio mismatch between fiber and matrix phases.

KEYWORDS: Micromechanics, Piezoelectric Composites & Structural Mechanics

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Nomenclature

Symbol	Description	Unit
α_f	Area of cross-section of original fiber	m^2
α_i^f $(i = 1\ to\ 4)$	Constants	----
α_i^m $(i = 1\ to\ 4)$	Constants	----
A_i $(i = 1\ to\ 3)$	Constants	----
B_i $(i = 1\ to\ 7)$	Constants	----
C	Stiffness	N/m^2
D	Dielectric displacement	C/m^2
e	Piezoelectric stress coefficient	C/m^2
E	Electric field	V/m
F_i $(i = 1\ to\ 4)$	Constants	----
g_i $(i = 1\ to\ 18)$	Experimental Constants	----
G_i $(i = 1\ to\ 4)$	Constants	----
i, j, k, l, m, n	Indices representing field quantities	----
I_i $(i = 1\ to\ 4)$	Constants	----
J_i $(i = 1\ to\ 4)$	Constants	----
R_{ij}	Ratio of piezoelectric stress coefficient of PFRC to piezoelectric fiber	----
σ	Stress	N/m^2
$\in$	Strain	--
κ	Dielectric Constant	C/Vm

1. INTRODUCTION

Piezoelectric materials have an excellent property of doing reciprocity of energy between the mechanical domain to electrical domain and vice versa. This makes piezoelectric ceramics such as PZT (Lead Zirconium Titanate) very attractive materials towards sensors and actuators applications. Piezoelectric materials have numerous applications in conventional as well as advanced engineering applications like aerospace, microphones, ultrasonic transducers, automatic control of remote explorations. These wide applications make piezoelectric materials a class of materials that requires attention for research and development toward miniaturisation of properties of these ceramics. While using in pure form, these materials have certain limitations, like low piezoelectric constants, shape control and high specific acoustic impedance, this can be overcome by combining these materials with non-piezoelectric materials. This results into piezoelectric composites bearing superior mechanical and electrical properties to the previous one.

The study of electromechanical properties of piezoelectric fibre reinforced composites (PFRC) has drawn a great deal of attention in recent years in form of mathematical modelling of behaviour of these composites. Various numerical methods as well as analytical methods has been used to describe the behaviour of such elements, though these methods have limitations of their own. Currently, there has been increasing research interest in developing analytical models to determine the effective electro-mechanical properties of such class of materials. In micromechanical approach, the overall properties of the composite are said to be volume average sum of the properties of a unit representative volume element (RVE) constituted of both fiber and matrix surrounding it. Effective properties can be derived from the results obtained by the analysis of the RVE. A number of such efforts for determining effective electro-mechanical properties of such composites [6-8] have been reported in the literature.

All of such models have many assumptions, which are common to all in order to simplify the complex calculations. In actual practice, these assumptions do have their own effect on final outcome. One of such assumption is that the effective coefficients have been calculated with respect to average electric field in the homogenized piezoelectric composite. In actual practice, there is a huge difference in electric field in fibre and matrix phases due to striking contrast in their dielectric properties. Authors [7-10] have used these same set of assumptions to predict overall effective properties of PFRC. Few improvements of properties have also been reported, by doing certain modification to earlier models. Kumar and Chakraborty [9] have extended study of Mallik and Ray to determine the overall thermo-electro-elastic properties, which is based on strength of materials approach.

Present work aims at developing a micromechanics-based model for the evaluation of effective electro-mechanical properties of PFRCs, which could be used in accurate analysis of PFRCs as actuators in smart applications. Micromechanics model for predicting the effective properties of the piezoelectric composite is based upon modified strength of materials and energy method. Results obtained has been compared to the effective properties obtained through strength of materials method [7-10].

2. MICROMECHANICS MODEL

Piezoelectric fibre reinforced ceramics consists of piezoelectric fibers aligned parallel to each other in longitudinal direction, surrounded by matrix phase. Bulk materials can be visualized as an assembly of a rectangular representative volume element (RVE) containing both fiber and matrix which can be repeated in three dimensions to obtain the bulk material. Figure 2 shows the transverse cross-section of such a representative volume element in a PFRC. This section

contains micromechanics modelling of such RVE with modified strength of material method. The effective coefficients obtained using modified strength element would be same as that of RVE in average sense.

For simplification, few assumptions have been made in this study. It is assumed here that the composite is subjected to a constant electric field in the direction transverse to that of fibers, electric field is same in both matrix and fiber. Piezoelectric composite is uniform throughout, behaves in linear elastic manner, and there is no slippage between fibers and matrix. Also, fibers are continuous and aligned parallel in longitudinal direction. Figure 1 shows a square array of fibers of circular cross-section and figure 2 gives representative volume element (RVE) of such an array. The circular cross-section of fiber is converted into square with equivalent area, and the RVE is divided into sub regions a shown in figure 2. Area of cross-section of original fiber is a_f, which can be modified to an equivalent square fiber of side s_f. Since length of side of RVE is s, the cross-section area of RVE becomes s^2. As cross-section area of circular fiber is equal to cross-section area equivalent square fiber i.e. $s_f^2 = a_f$ or $s_f = \sqrt{a_f}$. Fiber volume fraction for longitudinal fiber reinforced composite is the ratio of cross-section area of the fiber to cross-section area of RVE i.e.

$$v_f = \frac{a_f}{s^2} = \frac{s_f^2}{s^2} \tag{1}$$

$$\text{or, } \frac{s_f}{s} = \sqrt{v_f} \tag{2}$$

As observed from figure 2 matrix dimension is $s_m = s - s_f$, from eqn. (2) it can be written as

$$\frac{s_m}{s} = 1 - \sqrt{v_f} \tag{3}$$

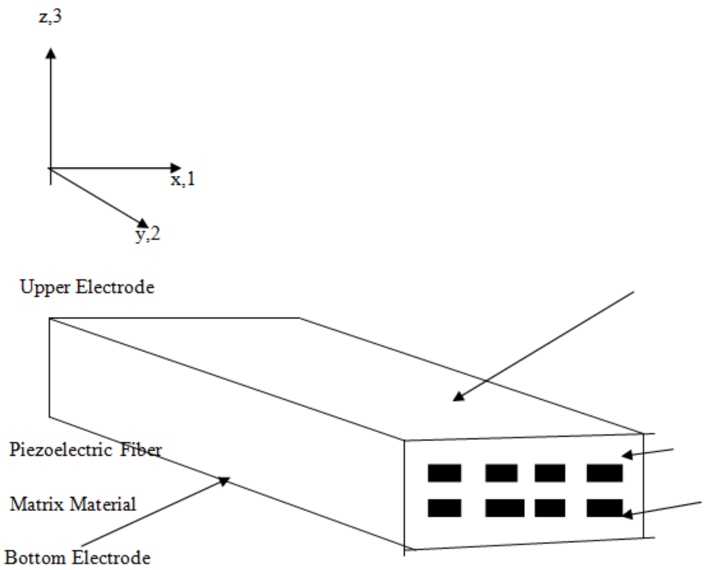

Figure 1: Schematic Diagram of PFRC Lamina.

Figure 2 illustrates that RVE can be subdivided into sub regions I and II, region I is matrix dominated while II is a composite consisting both fiber and matrix phases. In this approach, firstly effective properties of sub region II are computed as series connection of fiber and matrix subjected to transverse normal stress, which is followed by calculation of overall effective properties taking sub region I and II as parallel connection subjected to transverse normal stress.

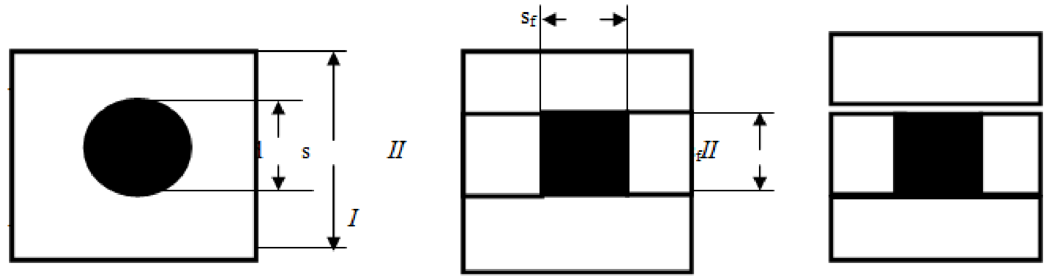

Figure 2: Transverse Cross-section of a Representative Volume Element of PFRC.

Constitutive equations for transversely isotropic piezoelectric active fibers in Voigt's two index notation are [7-10].

$$\begin{Bmatrix} \sigma_{11}^f \\ \sigma_{22}^f \\ \sigma_{33}^f \\ \sigma_{23}^f \\ \sigma_{31}^f \\ \sigma_{12}^f \end{Bmatrix} = \begin{bmatrix} C_{11}^f & C_{12}^f & C_{13}^f & 0 & 0 & 0 \\ C_{12}^f & C_{22}^f & C_{23}^f & 0 & 0 & 0 \\ C_{13}^f & C_{23}^f & C_{33}^f & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^f & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^f & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^f \end{bmatrix} \begin{Bmatrix} \epsilon_{11}^f \\ \epsilon_{22}^f \\ \epsilon_{33}^f \\ \epsilon_{23}^f \\ \epsilon_{31}^f \\ \epsilon_{12}^f \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31}^f \\ 0 & 0 & e_{32}^f \\ 0 & 0 & e_{33}^f \\ 0 & e_{24}^f & 0 \\ e_{15}^f & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1^f \\ E_2^f \\ E_3^f \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} D_1^f \\ D_2^f \\ D_3^f \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15}^f & 0 \\ 0 & 0 & 0 & e_{24}^f & 0 & 0 \\ e_{31}^f & e_{32}^f & e_{33}^f & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_{11}^f \\ \epsilon_{22}^f \\ \epsilon_{33}^f \\ \epsilon_{23}^f \\ \epsilon_{31}^f \\ \epsilon_{12}^f \end{Bmatrix} + \begin{bmatrix} \kappa_{11}^f & 0 & 0 \\ 0 & \kappa_{22}^f & 0 \\ 0 & 0 & \kappa_{33}^f \end{bmatrix} \begin{Bmatrix} E_1^f \\ E_2^f \\ E_3^f \end{Bmatrix} \tag{5}$$

The constitutive equations for isotropic passive matrix phases in Voigt's two index notation are

$$\begin{Bmatrix} \sigma_{11}^m \\ \sigma_{22}^m \\ \sigma_{33}^m \\ \sigma_{23}^m \\ \sigma_{31}^m \\ \sigma_{12}^m \end{Bmatrix} = \begin{bmatrix} C_{11}^m & C_{12}^m & C_{13}^m & 0 & 0 & 0 \\ C_{12}^m & C_{22}^m & C_{23}^m & 0 & 0 & 0 \\ C_{13}^m & C_{23}^m & C_{33}^m & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}^m & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55}^m & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}^m \end{bmatrix} \begin{Bmatrix} \epsilon_{11}^m \\ \epsilon_{22}^m \\ \epsilon_{33}^m \\ \epsilon_{23}^m \\ \epsilon_{31}^m \\ \epsilon_{12}^m \end{Bmatrix}$$

$$\begin{Bmatrix} D_1^m \\ D_2^m \\ D_3^m \end{Bmatrix} = \begin{bmatrix} \kappa_{11}^m & 0 & 0 \\ 0 & \kappa_{22}^m & 0 \\ 0 & 0 & \kappa_{33}^m \end{bmatrix} \begin{Bmatrix} E_1^m \\ E_2^m \\ E_3^m \end{Bmatrix} \tag{6}$$

here C_{ij}^x, e_{ij}^x, κ_{ij}^x are elastic constants, piezoelectric constants and dielectric constants, respectively and σ_{ij}^x, ϵ_{kl}^x, E_k^x, D_i^x are the stress, strain, electric field, electric displacement, respectively. Superscript x denotes piezoelectric fiber (f) or matrix (m) phases, respectively.

The resultant constitutive equations of composite can be written as

$$\begin{Bmatrix}\sigma_{11}^c\\ \sigma_{22}^c\\ \sigma_{33}^c\\ \sigma_{23}^c\\ \sigma_{31}^c\\ \sigma_{12}^c\end{Bmatrix}=\begin{bmatrix}C_{11}^c & C_{12}^c & C_{13}^c & 0 & 0 & 0\\ C_{12}^c & C_{22}^c & C_{23}^c & 0 & 0 & 0\\ C_{13}^c & C_{23}^c & C_{33}^c & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44}^c & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55}^c & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}^c\end{bmatrix}\begin{Bmatrix}\epsilon_{11}^c\\ \epsilon_{22}^c\\ \epsilon_{33}^c\\ \epsilon_{23}^c\\ \epsilon_{31}^c\\ \epsilon_{12}^c\end{Bmatrix}-\begin{bmatrix}0 & 0 & e_{31}^c\\ 0 & 0 & e_{32}^c\\ 0 & 0 & e_{33}^c\\ 0 & e_{24}^c & 0\\ e_{15}^c & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_1^c\\ E_2^c\\ E_3^c\end{Bmatrix}$$

$$\begin{Bmatrix}D_1^c\\ D_2^c\\ D_3^c\end{Bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 & e_{15}^c & 0\\ 0 & 0 & 0 & e_{24}^c & 0 & 0\\ e_{31}^c & e_{32}^c & e_{33}^c & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\epsilon_{11}^c\\ \epsilon_{22}^c\\ \epsilon_{33}^c\\ \epsilon_{23}^c\\ \epsilon_{31}^c\\ \epsilon_{12}^c\end{Bmatrix}+\begin{bmatrix}\kappa_{11}^c & 0 & 0\\ 0 & \kappa_{22}^c & 0\\ 0 & 0 & \kappa_{33}^c\end{bmatrix}\begin{Bmatrix}E_1^c\\ E_2^c\\ E_3^c\end{Bmatrix} \tag{7}$$

Where, C_{ij}^c, e_{ij}^c, κ_{ij}^c are to be determined in order to evaluate complete electro-mechanical behaviour of the composite.

Considering series connection of fiber and matrix in sub region II subjected to transverse loading, geometric compatibility requires

$$\delta_{22}^{II}=\delta_{22}^{IIf}+\delta_{22}^{IIm} \tag{8}$$

δ_{22}^{IIf}, δ_{22}^{IIm} are displacements in fibers and matrix when transverse loading in direction-2 is applied. Consequently,

$$\epsilon_{22}^{II}s=\epsilon_{22}^{f}s_f+\epsilon_{22}^{m}s_m \tag{9}$$

Using Eqns. (1), (2) Eqn. (9) can be written as

$$\epsilon_{22}^{II}=\sqrt{v_f}\epsilon_{22}^{f}+(1-\sqrt{v_f})\epsilon_{22}^{m} \tag{10}$$

Similarly, for direction-3, the relation can be written as

$$\epsilon_{33}^{II}=\sqrt{v_f}\epsilon_{33}^{f}+(1-\sqrt{v_f})\epsilon_{33}^{m} \tag{11}$$

As perfect bonding is assumed, therefore the strains are same in direction-2

$$\epsilon_{11}^{II}=\epsilon_{11}^{f}=\epsilon_{11}^{m} \tag{12}$$

Also, an electric field is applied in transverse direction only and is same in both fiber and matrix for sub region II

$$E_3^{II}=E_3^{f}=E_3^{m} \tag{13}$$

Stresses in fiber and matrix phases in transverse directions 2 and 3 are equal and written as

$$\sigma_{22}^{f}=\sigma_{22}^{m} \tag{14}$$

$$\sigma_{33}^{f}=\sigma_{33}^{m} \tag{15}$$

Using Eqns. (4), (5), (10), (11), (12), and (13) into Eqn. (14) and (15) the following Relations are obtained

$$A_1\epsilon_{22}^{f}+A_2\epsilon_{33}^{f}=B_1\epsilon_{11}^{c}+B_2\epsilon_{22}^{c}+B_3\epsilon_{33}^{c}-B_4E_3^{c} \tag{16}$$

$$A_2\epsilon_{22}^f + A_3\epsilon_{33}^f = B_5\epsilon_{11}^c + B_3\epsilon_{22}^c + B_6\epsilon_{33}^c - B_7E_3^c \tag{17}$$

where,

$$A_1 = \left(1-\sqrt{v_f}\right)C_{22}^f + \sqrt{v_f}C_{22}^m$$

$$A_2 = \left(1-\sqrt{v_f}\right)C_{23}^f + \sqrt{v_f}C_{23}^m$$

$$A_3 = \left(1-\sqrt{v_f}\right)C_{33}^f + \sqrt{v_f}C_{33}^m$$

$$B_1 = \left(1-\sqrt{v_f}\right)(C_{12}^m - C_{12}^f)$$

$$B_2 = C_{22}^m$$

$$B_3 = C_{23}^m$$

$$B_4 = \left(1-\sqrt{v_f}\right)(e_{32}^m - e_{32}^f)$$

$$B_5 = \left(1-\sqrt{v_f}\right)(C_{13}^m - C_{13}^f)$$

$$B_6 = C_{33}^m$$

$$B_7 = \left(1-\sqrt{v_f}\right)(e_{33}^m - e_{33}^f) \tag{18}$$

On solving Eqns. (16) and (17), the following Eqns. are obtained

$$\epsilon_{22}^f = f_1\epsilon_{11}^c + f_2\epsilon_{22}^c + f_3\epsilon_{33}^c - f_4E_3^c \tag{19}$$

$$\epsilon_{33}^f = g_1\epsilon_{11}^c + g_2\epsilon_{22}^c + g_3\epsilon_{33}^c - g_4E_3^c \tag{20}$$

where,

$$G_1 = \frac{a_2b_1-a_1b_5}{a_2^2-a_1a_3}$$

$$G_2 = \frac{a_2b_2-a_1b_3}{a_2^2-a_1a_3}$$

$$G_3 = \frac{a_2b_3-a_1b_6}{a_2^2-a_1a_3}$$

$$G_4 = \frac{a_2b_4-a_1b_7}{a_2^2-a_1a_3}$$

$$F_1 = \frac{b_1-a_2g_1}{a_1}$$

$$F_2 = \frac{b_2-a_2g_2}{a_1}$$

$$F_3 = \frac{b_3-a_2g_3}{a_1}$$

$$F_4 = \frac{b_4-a_2g_4}{a_1} \tag{21}$$

Using Eqns. (4), (5), (11), (12), (13) into Eqns. (19) and (20) transverse stresses in 2-direction can be expressed as

$$\sigma_{22}^{f} = l_1\epsilon_{11}^{c} + l_2\epsilon_{22}^{c} + l_3\epsilon_{33}^{c} - l_4E_3^{c} \tag{22}$$

$$\sigma_{22}^{m} = m_1\epsilon_{11}^{c} + m_2\epsilon_{22}^{c} + m_3\epsilon_{33}^{c} - m_4E_3^{c} \tag{23}$$

Where,

$$I_1 = C_{12}^{f} + C_{22}^{f}f_1 + C_{23}^{f}g_1$$

$$I_2 = C_{22}^{f}f_2 + C_{23}^{f}g_2$$

$$I_3 = C_{22}^{f}f_3 + C_{23}^{f}g_3$$

$$I_4 = C_{22}^{f}f_4 + C_{23}^{f}g_4$$

and

$$J_1 = C_{12}^{m} + \frac{\sqrt{v_f}C_{22}^{m}}{1-\sqrt{v_f}}f_1 - \frac{\sqrt{v_f}C_{23}^{m}}{1-\sqrt{v_f}}g_1$$

$$J_2 = \frac{C_{22}^{m}}{1-\sqrt{v_f}}f_1 + \frac{\sqrt{v_f}C_{22}^{m}}{1-\sqrt{v_f}}f_2 - \frac{\sqrt{v_f}C_{23}^{m}}{1-\sqrt{v_f}}g_2$$

$$J_3 = \frac{C_{23}^{m}}{1-\sqrt{v_f}}f_1 + \frac{\sqrt{v_f}C_{22}^{m}}{1-\sqrt{v_f}}f_3 - \frac{\sqrt{v_f}C_{23}^{m}}{1-\sqrt{v_f}}g_3$$

$$J_4 = e_{32}^{m} + \frac{\sqrt{v_f}C_{22}^{m}}{1-\sqrt{v_f}}f_4 - \frac{\sqrt{v_f}C_{23}^{m}}{1-\sqrt{v_f}}g_4$$

Subsequently, considering sub regions I and II as combined unit in RVE and applying following conditions

$$\sigma_{22}^{c} = \sqrt{v_f}\sigma_{22}^{f} + (1-\sqrt{v_f})\sigma_{22}^{m} \tag{24}$$

$$\sigma_{33}^{c} = \sqrt{v_f}\sigma_{33}^{f} + (1-\sqrt{v_f})\sigma_{33}^{m} \tag{25}$$

The effective stiffness and piezoelectric coefficients obtained as

$$C_{11}^{c} = \sqrt{v_f}C_{11}^{f} + \left(1-\sqrt{v_f}\right)C_{11}^{m} + \sqrt{v_f}\left(C_{12}^{f} - C_{12}^{m}\right)f_1 + \sqrt{v_f}\left(C_{13}^{f} - C_{13}^{m}\right)g_1$$

$$C_{12}^{c} = C_{12}^{m} + \sqrt{v_f}\left(C_{12}^{f} - C_{12}^{m}\right)f_2 + \sqrt{v_f}\left(C_{13}^{f} - C_{13}^{m}\right)g_2$$

$$C_{13}^{c} = C_{13}^{m} + \sqrt{v_f}\left(C_{12}^{f} - C_{12}^{m}\right)f_3 + \sqrt{v_f}\left(C_{13}^{f} - C_{13}^{m}\right)g_3$$

$$C_{22}^{c} = \sqrt{v_f}l_2 + (1-\sqrt{v_f})m_2$$

$$C_{23}^{c} = \sqrt{v_f}l_3 + (1-\sqrt{v_f})m_3$$

$$C_{33}^{c} = C_{23}^{f}f_3 + C_{33}^{f}g_3$$

$$e_{31}^{c} = \sqrt{v_f}e_{31}^{f} + \left(1-\sqrt{v_f}\right)e_{31}^{m} + \sqrt{v_f}\left(C_{12}^{f} - C_{12}^{m}\right)f_4 + \sqrt{v_f}\left(C_{13}^{f} - C_{13}^{m}\right)g_4$$

$$e_{32}^{c} = \sqrt{v_f}l_4 + (1-\sqrt{v_f})m_4$$

$$e_{33}^{c} = e_{33}^{f} + C_{23}^{f} f_4 + C_{33}^{f} g_4$$

3. ENERGY METHOD

Energy functions for piezoelectric composite, fiber and matrix are given by

$$U^{c} = \int \left\{ -\frac{1}{2}\left({\epsilon^{c}}^{T} {C^{c}}^{T} \epsilon^{c} + 2{E^{c}}^{T} {e^{c}}^{T} \epsilon^{c} \right) + \frac{1}{2} {E^{c}}^{T} {\epsilon^{c}}^{T} E^{c} \right\} dV \tag{26}$$

$$U^{f} = \int \left\{ -\frac{1}{2}\left({\epsilon^{f}}^{T} {C^{f}}^{T} \epsilon^{f} + 2{E^{f}}^{T} {e^{f}}^{T} \epsilon^{f} \right) + \frac{1}{2} {E^{f}}^{T} {\epsilon^{f}}^{T} E^{f} \right\} dV \tag{27}$$

$$U^{m} = \int \left\{ -\frac{1}{2}\left({\epsilon^{m}}^{T} {C^{m}}^{T} \epsilon^{m} + 2{E^{m}}^{T} {e^{m}}^{T} \epsilon^{m} \right) + \frac{1}{2} {E^{m}}^{T} {\epsilon^{m}}^{T} E^{m} \right\} dV \tag{28}$$

Under the given state of stress, total energy stored in composite is equal to sum of energies stored in fibers and matrix. Thus,

$$U^{c} = U^{f} + U^{m} \tag{29}$$

Stresses and strains in fibers and matrix are defined in terms of corresponding composite quantities, as given below

$$\sigma_{11}^{f} = g_1 \sigma_{11}^{c} \tag{30}$$

$$\sigma_{11}^{m} = g_2 \sigma_{11}^{c} \tag{31}$$

$$\epsilon_{22}^{f} = g_3 \epsilon_{22}^{c} \tag{32}$$

$$\epsilon_{22}^{m} = g_4 \epsilon_{22}^{c} \tag{33}$$

$$\epsilon_{33}^{f} = g_5 \epsilon_{33}^{c} \tag{34}$$

$$\epsilon_{33}^{m} = g_6 \epsilon_{33}^{c} \tag{35}$$

$$\epsilon_{23}^{f} = g_7 \epsilon_{23}^{c} \tag{36}$$

$$\epsilon_{23}^{m} = g_8 \epsilon_{23}^{c} \tag{37}$$

$$\epsilon_{31}^{f} = g_9 \epsilon_{31}^{c} \tag{38}$$

$$\epsilon_{31}^{m} = g_{10} \epsilon_{31}^{c} \tag{39}$$

$$\epsilon_{12}^{f} = g_{11} \epsilon_{12}^{c} \tag{40}$$

$$\epsilon_{12}^{m} = g_{12} \epsilon_{12}^{c} \tag{41}$$

$$E_{1}^{f} = g_{13} E_{1}^{c} \tag{42}$$

$$E_{1}^{m} = g_{14} E_{1}^{c} \tag{43}$$

$$E_{2}^{f} = g_{15} E_{2}^{c} \tag{44}$$

$$E_{2}^{m} = g_{16} E_{2}^{c} \tag{45}$$

$$E_{3}^{f} = g_{17} E_{3}^{c} \tag{46}$$

$$E_3^m = g_{18} E_3^c \tag{47}$$

Using Eqns. (30)-(35), (46) and (47) following relations are obtained

$$\epsilon_{11}^f = a_1^f \epsilon_{11}^c + a_2^f \epsilon_{22}^c + a_3^f \epsilon_{33}^c - a_4^f E_3^c \tag{48}$$

$$\epsilon_{11}^m = a_1^m \epsilon_{11}^c + a_2^m \epsilon_{22}^c + a_3^m \epsilon_{33}^c - a_4^m E_3^c \tag{49}$$

where,

$$a_1^f = \frac{g_1 C_{11}^c}{C_{11}^f}$$

$$a_1^m = \frac{g_2 C_{11}^c}{C_{11}^m}$$

$$a_2^f = \frac{(g_1 C_{12}^c - g_3 C_{12}^f)}{C_{11}^f}$$

$$a_2^m = \frac{(g_2 C_{12}^c - g_4 C_{12}^m)}{C_{11}^f}$$

$$a_3^f = \frac{(g_1 C_{13}^c - g_5 C_{13}^f)}{C_{11}^f}$$

$$a_3^m = \frac{(g_2 C_{13}^c - g_6 C_{13}^f)}{C_{11}^f}$$

$$a_4^f = \frac{(g_1 e_{31}^c - g_{17} e_{31}^m)}{C_{11}^f}$$

$$a_4^m = \frac{(g_2 e_{31}^c - g_{18} e_{31}^m)}{C_{11}^f} \tag{50}$$

Using Eqns. (30)-(49) into Eqn. (29), equating the coefficients of like terms both side, effective coefficients are obtained.

$$C_{11}^c = \frac{C_{11}^f C_{11}^m}{g_1^2 C_{11}^m + g_2^2 C_{11}^f}$$

$$C_{12}^c = a_1^f g_3 C_{12}^f + a_1^m g_4 C_{12}^m$$

$$C_{13}^c = a_1^f g_5 C_{13}^f + a_1^m g_6 C_{13}^m$$

$$C_{22}^c = \frac{\left(g_1 C_{12}^c - g_3 C_{12}^f\right)^2}{C_{11}^f} + \frac{\left(g_2 C_{12}^c - g_4 C_{12}^f\right)^2}{C_{11}^m} + g_3^2 C_{22}^f + g_4^2 C_{22}^m + 2a_2^f g_3 C_{12}^f + 2a_2^m g_4 C_{12}^m$$

$$C_{23}^c = a_3^f g_3 C_{12}^f + a_3^m g_4 C_{12}^m + a_2^f g_5 C_{13}^f + a_2^m g_6 C_{13}^m + g_3 g_5 C_{23}^f + g_4 g_6 C_{23}^m$$

$$C_{33}^c = \frac{\left(g_1 C_{12}^c - g_5 C_{12}^f\right)^2}{C_{11}^f} + \frac{\left(g_2 C_{12}^c - g_6 C_{12}^f\right)^2}{C_{11}^m} + g_5^2 C_{33}^f + g_6^2 C_{33}^m + 2a_3^f g_5 C_{13}^f + 2a_3^m g_6 C_{13}^m$$

$$e_{31}^c = a_1^f g_{17} e_{31}^f + a_1^m g_{18} e_{31}^m$$

$$e_{32}^{c} = a_2^f g_{17} e_{31}^f + a_2^m g_{18} e_{31}^m + g_3 g_{17} e_{32}^f + g_4 g_{18} e_{32}^m + a_4^f g_3 C_{12}^f + a_4^m g_4 C_{12}^m$$

$$e_{33}^{c} = a_3^f g_{17} e_{31}^f + a_3^m g_{18} e_{31}^m + g_5 g_{17} e_{33}^f + g_6 g_{18} e_{33}^m + a_4^f g_5 C_{13}^f + a_4^m g_6 C_{13}^m$$

It should be noted that constants g_i $(i = 1,2,3 \ldots \ldots 18)$ are experimental parameters and must be obtained through experimental techniques. However, experimental determination of these parameters is out of the scope for the present study. Thus, these parameters are obtained with the theoretical assumptions of modified strength of materials approach as discussed in the previous section.

4. RESULTS AND DISCUSSIONS

Above derived equations obtained through modified strength of materials and energy methods has been used to study the diversion of effective properties of PFRC, obtained by strength of materials method [7-10]. Table 1-2 shows the elastic, electric and dielectric properties of the constituents i.e fiber and matrix phases.

Table 1: Elastic Properties of Piezoelectric Fibers and Matrix [9,10]

Material	C_{11}(GPa)	C_{12}(GPa)	C_{13}(GPa)	C_{33}(GPa)	C_{44}(GPa)	C_{66}(GPa)
PZT-5	121	75.4	75.2	111	22.8	21.1
Epoxy	3.86	2.57	2.57	3.86	0.64	0.64

Table 2: Piezoelectric and Dielectric Properties of Fibers and Matrix[9-10]

Material	e_{31}(C/m^2)	e_{33}(C/m^2)	e_{15}(C/m^2)	κ_{11}(10^{-9} C/Vm)	κ_{33}(10^{-9} C/Vm)
PZT-5	-5.4	15.8	12.3	8.11	7.35
Epoxy	0	0	0	0.0797	0.0797

The study carried here provides a comparative study of variation in effective properties as a function of fiber volume fraction and matrix properties among strength of materials, modified strength of materials and energy method. The following non-dimensional parameters have been used in the analysis.

$$R_{31} = \frac{e_{31}^c}{e_{31}^f}$$

$$R_{32} = \frac{e_{32}^c}{e_{32}^f}$$

$$R_{33} = \frac{e_{33}^c}{e_{33}^f}$$

Figures 3–6 show the comparative study of effective elastic coefficient of composite with respect to change in fiber volume fraction. Results clearly shows that modified strength of method and energy method are close in values, while strength of materials overestimates the effective coefficient C_{11} while it underestimates effective coefficient C_{12}. Figure 5 shows effective coefficient estimated from strength of materials method is in close proximity with modified strength method and energy method. Again, a huge underestimation of effective coefficient C_{33} is reported as shown in figure 6. These wide deviations of effective properties occur due to fact that in strength of material approach, there has been certain assumptions which provide very poor results. In strength of materials approach [7-10] it is assumed that, equal strains are produced in fiber as well as matrix phases when stress is applied, while in actual practice these strains ought to be different. Modified strength of materials approach refines result for transverse stresses, while energy approach also gives better result, as Poisson’s ratio mismatch is tackled to a great degree through this model.

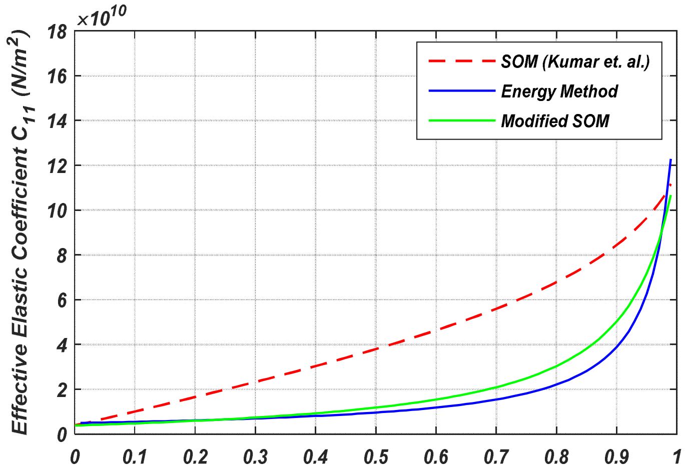

Figure 3: Variation of Elastic Coefficient C_{11} of PZT-5/Epoxy Composite with Fiber Volume Fraction.

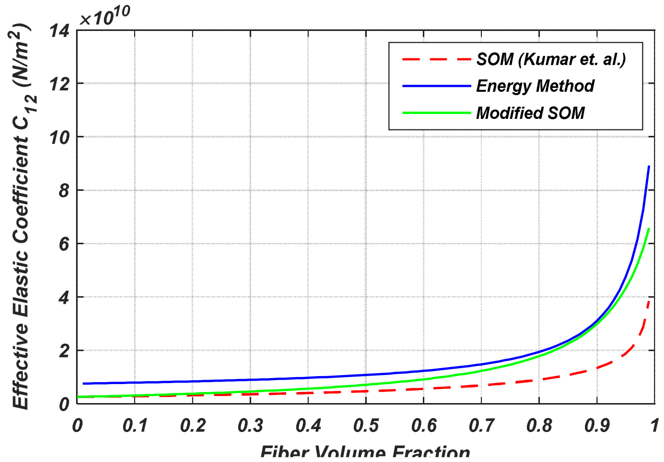

Figure 4: Variation of Elastic Coefficient C_{12} of PZT-5/Epoxy Composite with Fiber Volume Fraction.

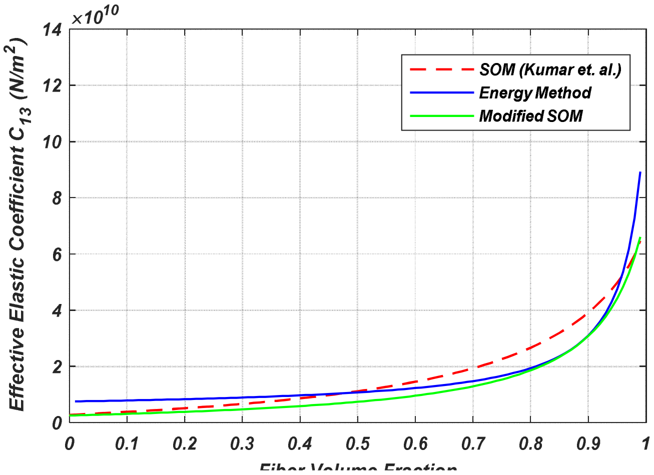

Figure 5: Variation of Elastic Coefficient C_{13} of PZT-5/Epoxy Composite with Fiber Volume Fraction.

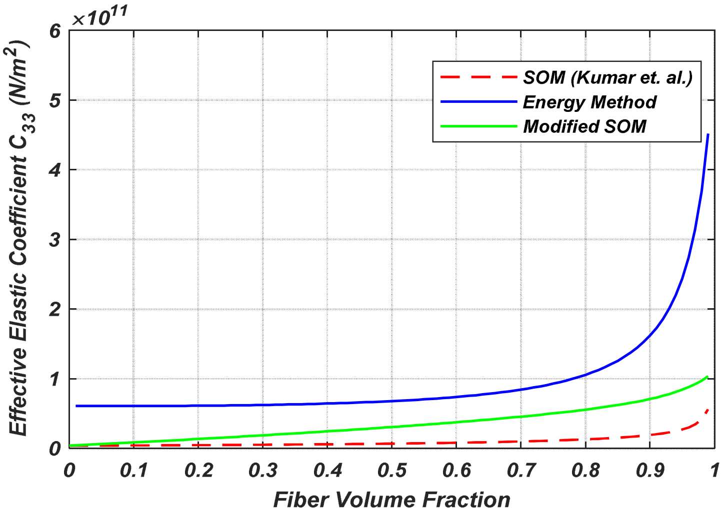

Figure 6: Variation of Elastic Coefficient C_{33} of PZT-5/Epoxy Composite with Fiber Volume Fraction.

Similar trends of overestimation and underestimation is also reported for R_{31}, R_{32}, and R_{33} for strength of materials in comparison with modified strength of materials and energy approach. Figure 7 shows value of ratio of piezoelectric coefficient R_{31} which has been highly overestimated in strength of materials method [7-10] as compared with modified strength of materials and energy method. While results shown in figure 8 Indicates the values of R_{32} are somewhat in good agreement with each other, at extremely lower and higher values of fiber volume fraction. In mid ranges of fiber volume fraction, there is symmetric divergence of properties between these three methods. In this case, strength of materials approach and modified strength of materials approach forms lower and upper bound of energy approach, respectively. Figure 9 shows ratio R_{33} is in good agreement for strength of materials method and energy method, while there is a divergence of modified strength of materials results from these two methods.

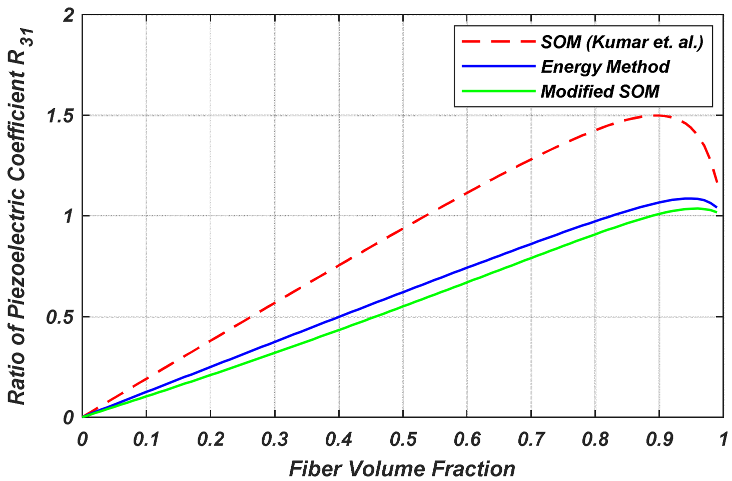

Figure 7: Variation of Ratio of Piezoelectric Coefficient $-e^c_{31}/e^f_{31}$ PZT-5/Epoxy Composite with Fiber Volume Fraction.

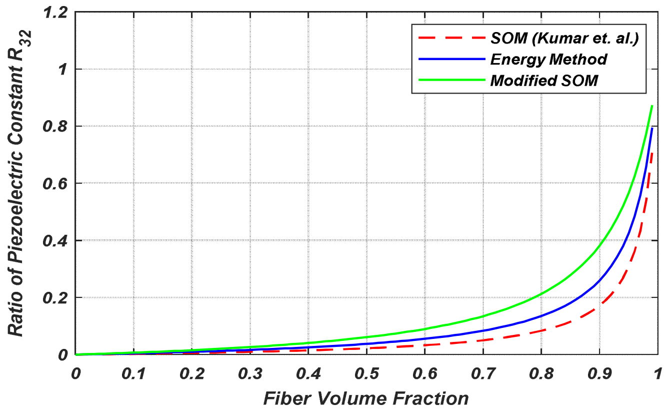

Figure 8: Variation of Ratio of Piezoelectric Coefficient e_{32}^{c}/e_{32}^{f} PZT-5/Epoxy Composite with Fiber Volume Fraction.

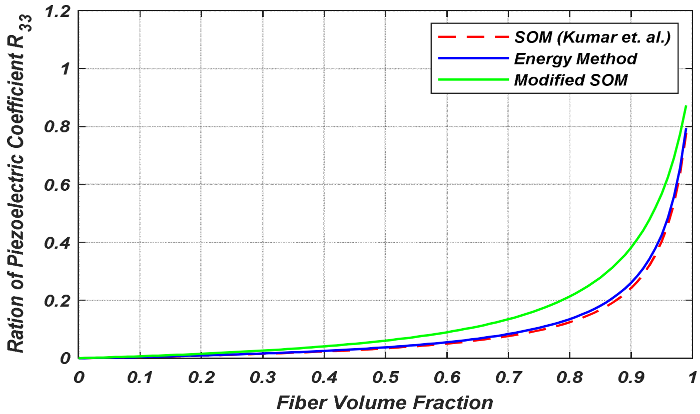

Figure 9: Variation of Ratio of Piezoelectric Coefficient e_{33}^{c}/e_{33}^{f} PZT-5/Epoxy Composite with Fiber Volume Fraction.

Though there is a huge variation in dielectric properties of the constituent phases of PFRC, figure 10 shows that the overall dielectric coefficient of composites is in good agreement with each other for all three methods i.e. strength of materials, modified strength of materials and energy method.

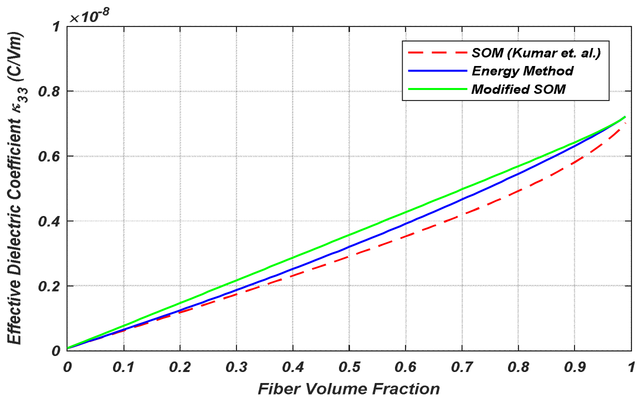

Figure 10: Variation of Dielectric Coefficient κ_{33}^{c} PZT-5/Epoxy Composite with Fiber Volume Fraction.

5. CONCLUSIONS

In this paper, two distinguished method of micromechanics i.e. modified strength of materials and energy method have been used to study the effective properties of piezo fiber reinforced composites. The results have been used to analyse deviation of effective properties estimated in earlier works [7-10] using strength of materials method. In various works [7-10], the strength of materials method models the problem on assumptions that the equal strains are produced in both fiber and matrix phases, while this assumption doesn't have significant effects on results while the longitudinal stress is applied to the composite. In case of transverse loading, the results obtained from strength of material would be far from accurate. Modified strength of materials method refines the result obtained from strength of materials method and provides a better estimated effective property. Energy method modelling of the problem eliminated the effect of Poisson's ratio mismatch between fiber and matrix phases. Thus, present study reveals better models for micromechanics analysis than the reported strength of materials methods for various coefficients, and expected to give more precise results than existing method.

It makes tailoring of properties of piezoelectric composites that contains piezoelectric fibers in piezoelectrically-inactive matrix, having superiors and improved effective electrical and mechanical properties. In addition, these models could be served as a benchmark for numerical analysis of PFRC problems and can be utilised to model coupled more piezoelectric composite problems, subjected to more complex boundary conditions.

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